KSU CET

S1 & S2 Notes

2019 Scheme

Manifest the leadership in you

19/9/2019

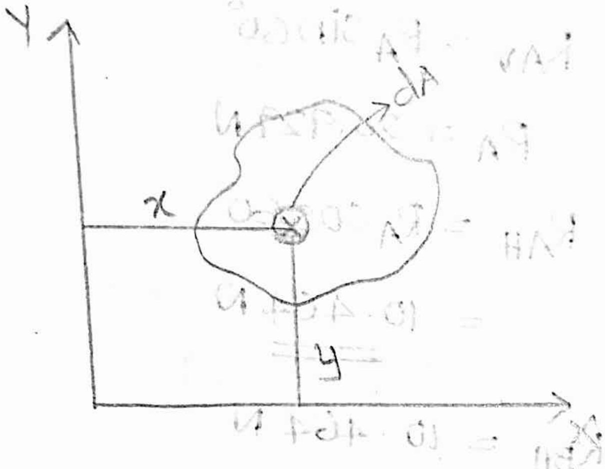
MODULE 3 : CENTROID

Centroid:

Centroid is the point at which total area assumed to be concentrated.

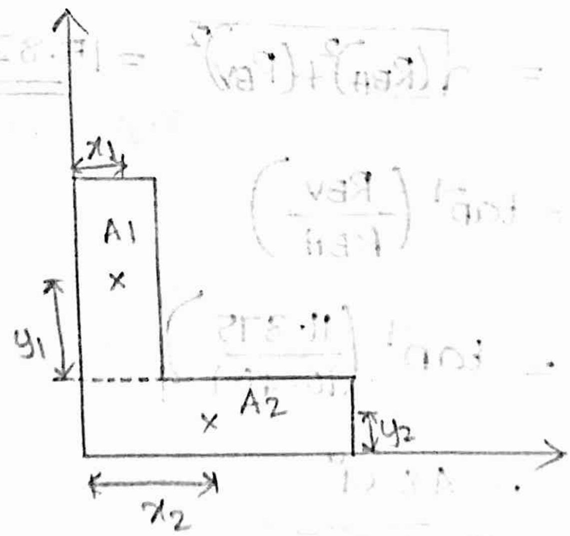
Centre of gravity:

Centre of gravity is the point at which the total weight is assumed to be concentrated.



$$\bar{x} = \frac{\int x\,dA}{\int dA}$$

$$\bar{y} = \frac{\int y\,dA}{\int dA}$$

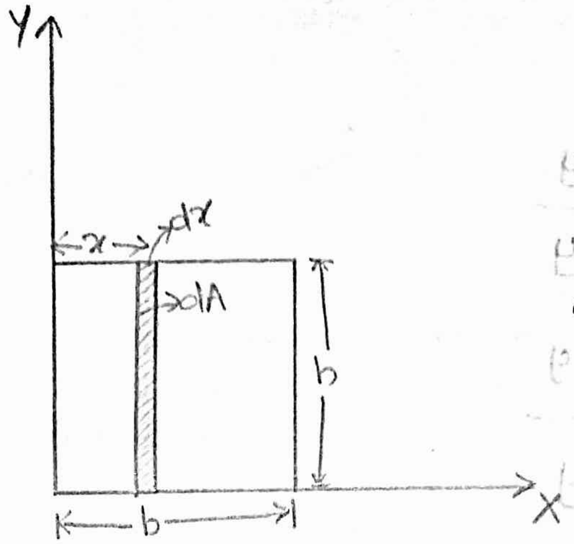


$$\bar{x} = \frac{A_1x_1 + A_2x_2}{A_1+A_2}$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2}{A_1+A_2}$$

Centroid of a Rectangle

To find $\bar{x}$: Consider a vertical strip of thickness dx at a distance x from the Y axis.



$$dA = h\,dx$$

$$\bar{x} = \frac{\int x\,dA}{\int dA}$$

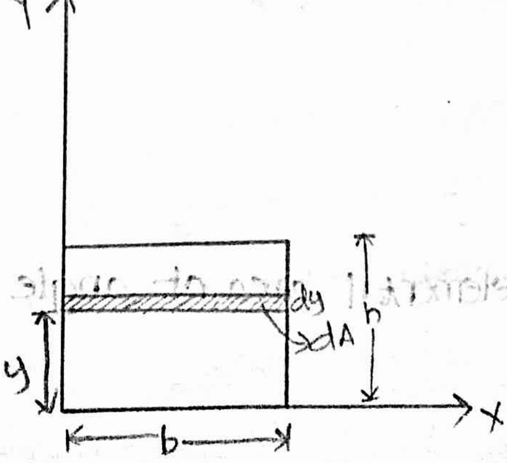
$$= \frac{\int x h\,dx}{\int h\,dx}$$

$$= \frac{\int_0^b x\,dx}{\int_0^b dx}$$

$$= \frac{\left[\frac{x^2}{2}\right]_0^b}{\left[x\right]_0^b} = \frac{b^2}{2b}$$

$$\bar{x} = \frac{b}{2}$$

To find $\bar{y}$: Consider a horizontal strip of thickness dy at a distance y from the x-axis.



$$dA = b\,dy$$

$$\bar{y} = \frac{\int y\,dA}{\int dA}$$

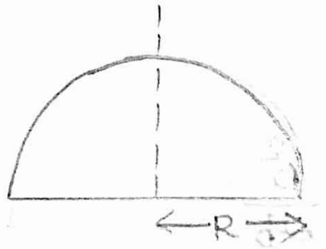
$$= \frac{\int y b\,dy}{\int b\,dy}$$

$$= \frac{\int_0^h y\,dy}{\int_0^h dy}$$

$$= \frac{h^2}{2 \times h} = \frac{h}{2}$$

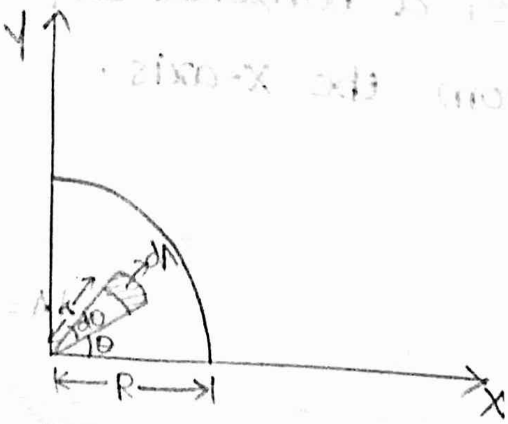
$\therefore$ Centroid of a rectangle $(\bar{x}, \bar{y}) = \left(\frac{b}{2}, \frac{h}{2}\right)$

Q: Locate the centroid of semicircle of radius 'R'.

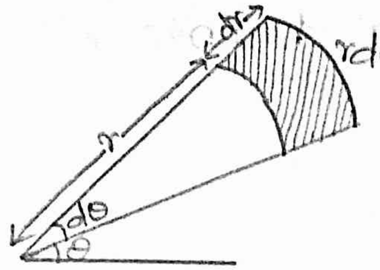


$$\bar{y} = \frac{\int y\,dA}{\int dA}$$

Consider a quadrant of a circle.



Consider a small elemental area at angle 'θ' and radius 'r'.



Since the element is small, we can consider it as a rectangular area.

$$\therefore dA = r\,d\theta \cdot dr$$

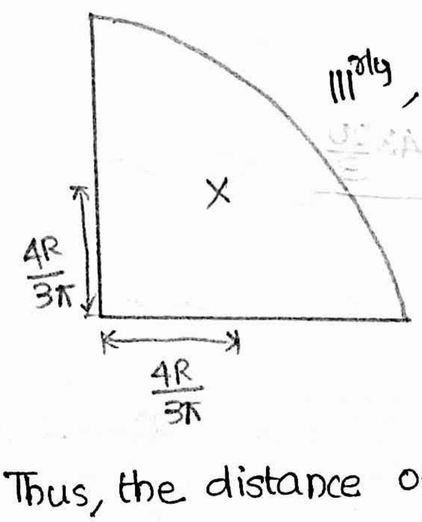
$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$\bar{x} = \frac{\int_{r=0}^{R}\int_{\theta=0}^{\pi/2} r\cos\theta \, . \, r\,d\theta\,dr}{\int_{0}^{R}\int_{0}^{\pi/2} r\,d\theta\,dr}$$

$$\bar{x} = \frac{\left[\frac{r^3}{3}\right]_0^R \left[\sin\theta\right]_0^{\pi/2}}{\left[\frac{r^2}{2}\right]_0^R \left[\theta\right]_0^{\pi/2}}$$

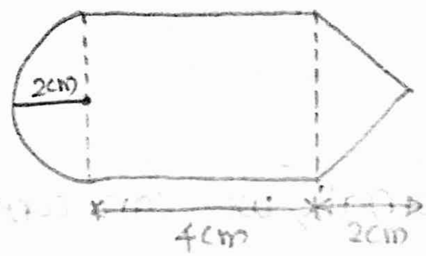
$$\bar{x} = \frac{\frac{R^3}{3}\times 1}{\frac{R^2}{2}\times\frac{\pi}{2}} = \frac{R^3}{3}\times\frac{4}{R^2\pi} = \frac{4R}{3\pi}$$

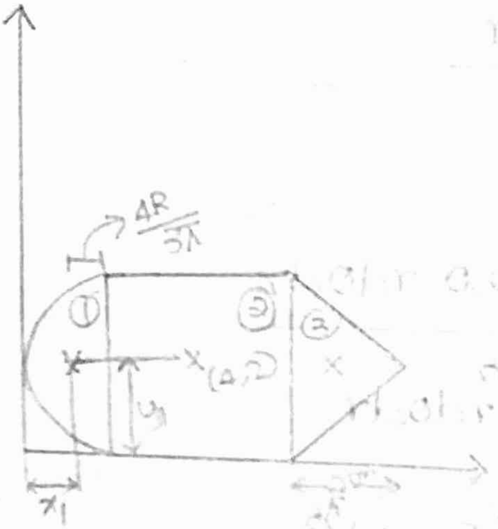


|||ly, $\bar{y} = \frac{4R}{3\pi}$

Thus, the distance of centroid of a semicircular lamina from its base is $\frac{4R}{3\pi}$

Q: Locate the centroid of area shown.





$x_1 = r - \frac{4r}{3\pi}$ $\qquad y_1 = r \qquad A_1 = \frac{\pi r^2}{2}$

$x_2 = 4 \qquad y_2 = 2 \qquad A_2 = a^2 = 4^2 = 16$

$x_3 = 6 + \frac{1}{3} \times 2 \qquad y_3 = \frac{4}{2} = 2 \qquad A_3 = \frac{1}{2} \times 4 \times 2 = 4$

$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1 + A_2 + A_3}$$

$$= \frac{2\pi \times 1.151 + 16 \times 4 + 4 \times \frac{20}{3}}{2\pi + 16 + 4}$$

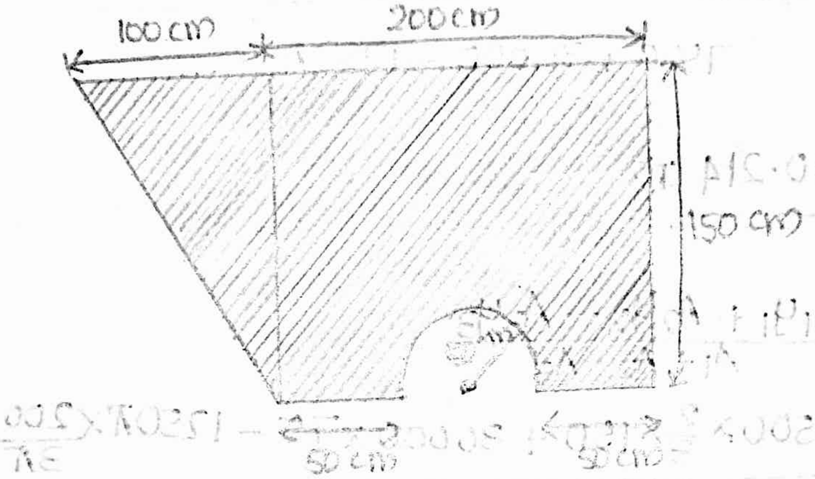
$$= \underline{\underline{3.725 \text{ cm}}}$$

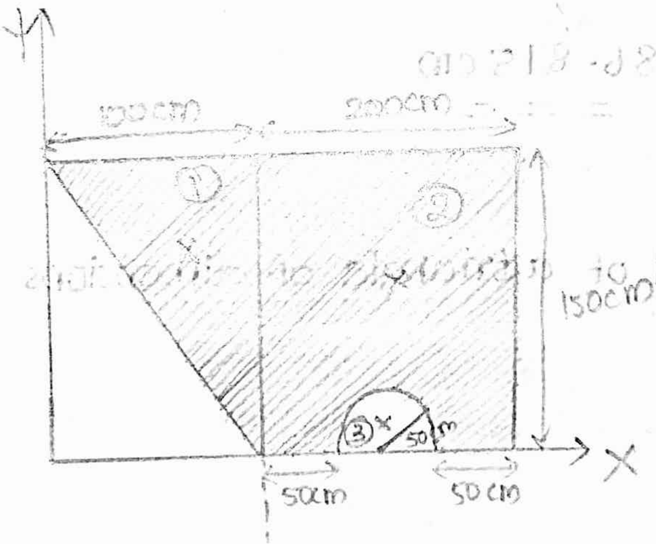
$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1 + A_2 + A_3}$$

$$= \frac{2\pi \times 2 + 16 \times 2 + 4 \times 2}{2\pi + 16 + 4} = \underline{\underline{2 \text{ cm}}}$$

centroid, $(\bar{x}, \bar{y}) = (3.725, 2)$

Q: Locate the centroid of area shown.





$$A_1 = \frac{1}{2} bh = \frac{1}{2} \times 100 \times 150 = 7500 \text{ cm}^2$$

$$A_2 = 200 \times 150 = 30000 \text{ cm}^2$$

$$A_3 = \pi \times \frac{50^2}{2} = \frac{2500\pi}{2} = 1250\pi \text{ cm}^2$$

$$x_1 = \frac{2}{3} \times 100 \text{ cm} \qquad y_1 = \frac{2}{3} \times 150 \text{ cm}$$

$$x_2 = 100 + 100 = 200 \text{ cm} \qquad y_2 = \frac{150}{2} = 75 \text{ cm}$$

$$x_3 = 100 + 100 = 200 \text{ cm} \qquad y_3 = \frac{4 \times 50}{3\pi} \text{ cm}$$

$$\bar{x} = \frac{A_1x_1 + A_2x_2 - A_3x_3}{A_1 + A_2 - A_3}$$

$$= \frac{7500 \times \frac{2}{3} \times 100 + 30000 \times 200 - 1250\pi \times 200}{7500 + 30000 - 1250\pi}$$

$$= 170.214 \text{ cm}$$

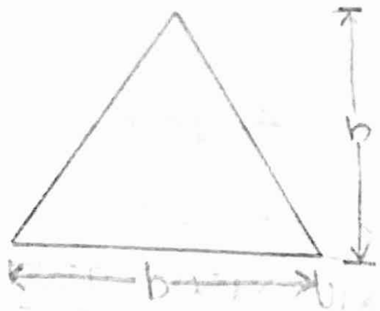
$$\bar{y} = \frac{A_1y_1 + A_2y_2 - A_3y_3}{A_1 + A_2 - A_3}$$

$$= \frac{7500 \times \frac{2}{3} \times 150 + 30000 \times 75 - 1250\pi \times \frac{200}{3\pi}}{7500 + 30000 - 1250\pi}$$

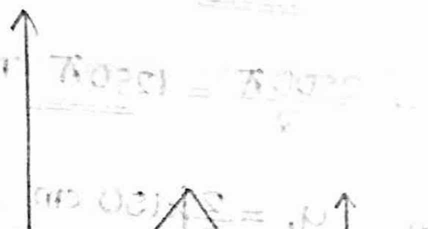
$$= 86.875 \text{ cm}$$

Home Work

Q: Locate the centroid of a triangle of dimensions b and h.



To find $\bar{y}$:



Consider an elemental strip of thickness dy at a distance y from x-axis

$$dA = b_1\,dy$$

Using the property of similar triangles,

$$\frac{h-y}{h} = \frac{b_1}{b}$$

$$b_1 = \frac{b}{h}(h-y)$$

$$dA = \frac{b}{h}(h-y)\,dy$$

$$\bar{y} = \frac{\int y\,dA}{\int dA} = \frac{\int_0^h y\frac{b}{h}(h-y)\,dy}{\int_0^h \frac{b}{h}(h-y)\,dy}$$

$$= \frac{\int_0^h (hy-y^2)\,dy}{\int_0^h (h-y)\,dy}$$

$$= \frac{\left[\frac{hy^2}{2} - \frac{y^3}{3}\right]_0^h}{\left[hy - \frac{y^2}{2}\right]_0^h}$$

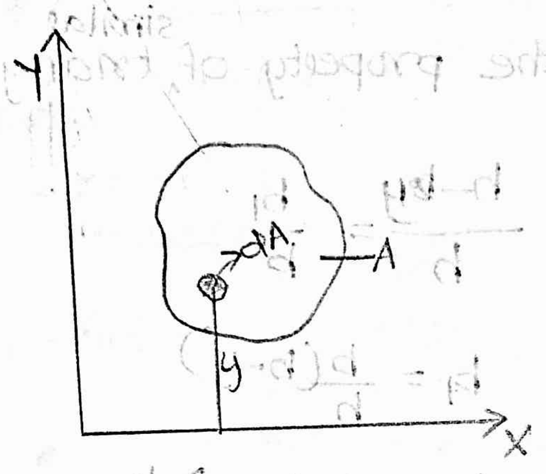
$$= \frac{\frac{h^3}{2} - \frac{h^3}{3}}{h^2 - \frac{h^2}{2}}$$

$$= h\left(\frac{\frac{1}{6}}{\frac{1}{2}}\right)$$

$$\bar{y} = \frac{h}{3}$$

MOMENT OF INERTIA

Moment of inertia is defined as the second moment of area about the axis.



$$I = y\,(y\,dA)$$

$$= y^2 dA$$

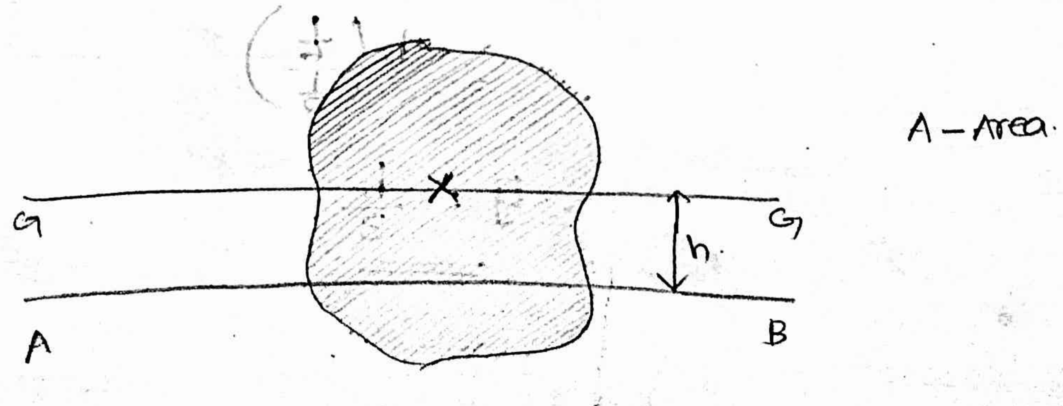
$$I = \int y^2 dA$$

PARALLEL AXIS THEOREM

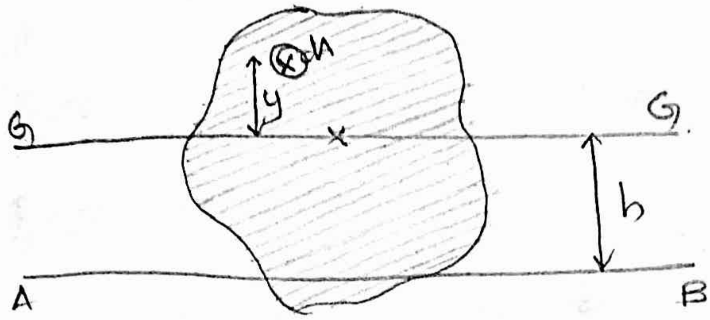
~~It states that if inertia about the cent~~

It states that if I_G is the moment of inertia of a plane lamina of area 'A' about its centroidal axis in the plane of the lamina, then, the moment of inertia about any axis AB which is parallel to the centroidal axis and at a distance 'h' from the centroidal axis is given by

$$\boxed{I_{AB} = I_G + Ah^2}$$



Proof:



Consider an elemental area dA at a distance 'y' from centroidal axis. Then, moment of area about AB is $(h+y)dA$ and second moment of area about AB is

$$I_{AB} = \int (h+y)^2 dA$$

$$= \int (h^2 + y^2 + 2hy) dA$$

$$= \int h^2 dA + \int y^2 dA + \int 2hy dA$$

$$= h^2 \int dA + \int y^2 dA + 2h \int y dA$$

$$= h^2 A + I_G + 0$$

$$\boxed{I_{AB} = I_G + Ah^2}$$

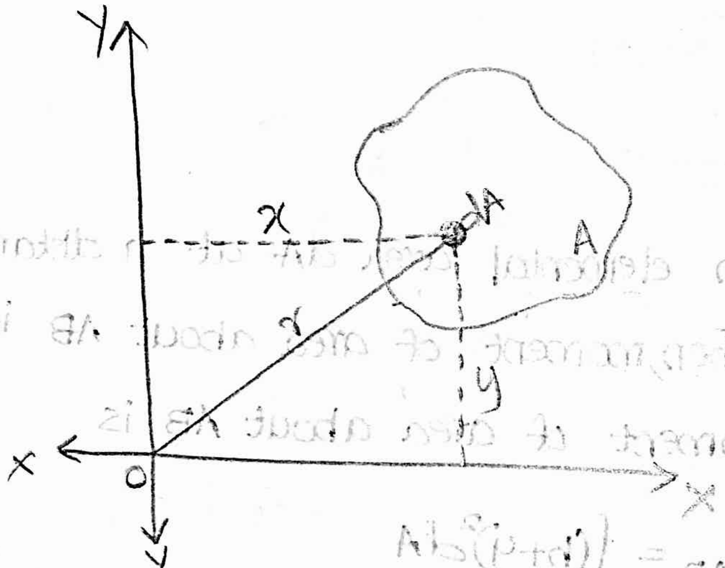
PERPENDICULAR AXIS THEOREM:

If I_{xx} and I_{xy} are moment of inertia of an area 'A' about mutually perpendicular axes XX and YY in the plane of the area, then moment of inertia of the area about the ZZ axis which is perpendicular to XX and YY and passing through the point of intersection of XX and YY is given by

$$\boxed{I_{ZZ} = I_{XX} + I_{YY}}$$

Proof:

Consider a small elemental area, dA at a distance 'r' from origin.



$$r^2 = x^2 + y^2$$

$$I_{ZZ} = \int r^2 dA$$

$$= \int (x^2+y^2) dA$$

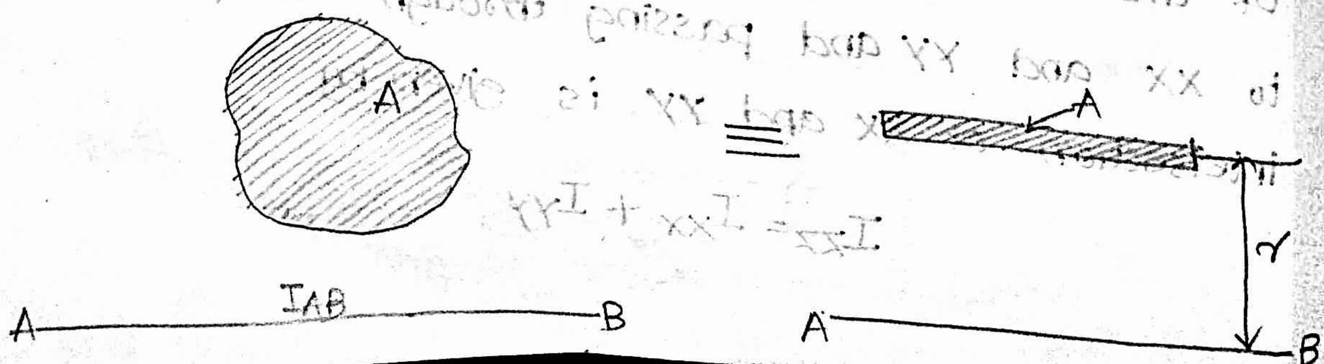
$$= \int x^2 dA + \int y^2 dA$$

$$\boxed{I_{ZZ} = I_{YY} + I_{XX}}$$

POLAR MOMENT OF INERTIA

Moment of inertia about ZZ axis which is perpendicular to XX and YY and passing through the point of intersection of XX and YY is called polar moment of inertia.

RADIUS OF GYRATION:



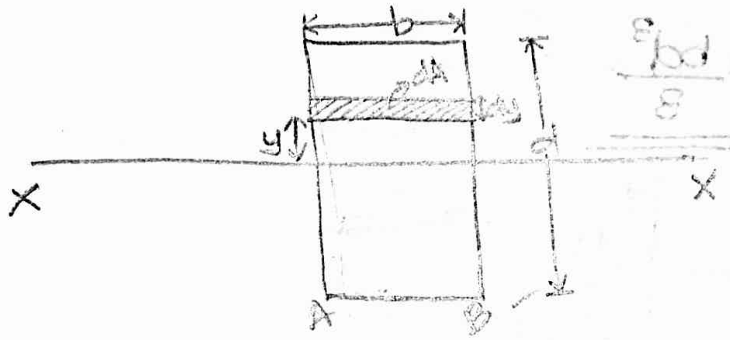
$$I_{AB} = Ar^2$$

where, r is called the radius of gyration.

MOMENT OF INERTIA OF REGULAR FIGURES

1. Rectangle

Consider a rectangle of dimensions 'b' and 'd'.



Consider a rectangular strip of thickness 'dy' at a distance 'y' from centroidal axis.

$$dA = b\,dy$$

$$I_{xx} = \int y^2 dA$$

$$= \int y^2 b\,dy$$

$$= b\int_{-d/2}^{d/2} y^2 dy$$

$$= b\left[\frac{y^3}{3}\right]_{-d/2}^{d/2}$$

$$= b\left[\frac{d^3}{24} + \frac{d^3}{24}\right]$$

$$= \frac{bd^3}{12}$$

$$I_{AB} = I_G + Ab^2$$

$$= \frac{bd^3}{12} + (bd)\left(\frac{d}{2}\right)^2$$

$$= \frac{bd^3}{12} + \frac{bd^3}{4}$$

$$= \frac{4bd^3}{12}$$

$$= \frac{bd^3}{3}$$

Consider a vertical strip of thickness dx at a distance 'x' from centroidal axis.

$$dA = d \cdot dx$$

$$I_{yy} = \int x^2 dA$$

$$= \int x^2 d\, dx$$

$$= d \int_{-b/2}^{b/2} x^2 dx$$

$$= d \left[\frac{x^3}{3}\right]_{-b/2}^{b/2}$$

$$= \frac{db^3}{12}$$

$$I_{AB} = I_G + Ab^2$$

$$= \frac{db^3}{12} + bd\left(\frac{b}{2}\right)^2$$

$$= \frac{db^3}{12} + \frac{db^3}{4}$$

$$= \frac{4b^3d}{12}$$

$$= \frac{db^3}{3}$$

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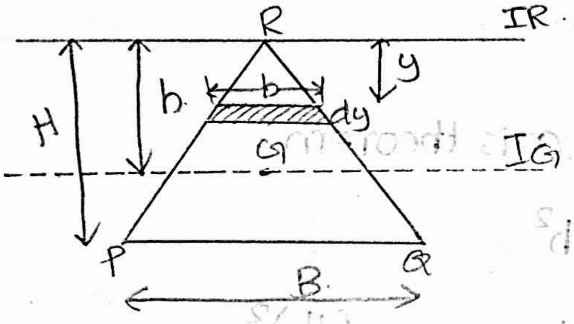
MOMENT OF INERTIA

(b) Moment of Inertia of a Triangle.



Consider an element of thickness dy at a distance 'y' from the vertex R.

Area of element, $dA = b\,dy$.

$$I_R = \int y^2\,dA$$

$$I_R = \int_0^H y^2 b\,dy$$

$$\frac{b}{B} = \frac{y}{H}$$

$$\therefore b = \frac{B}{H} \times y$$

$$I_R = \int_0^H y^2 \times \frac{B}{H}\,y\,dy$$

$$= \frac{B}{H}\left[\frac{y^4}{4}\right]_0^H$$

$$= \frac{B}{H}\left(\frac{H^4}{4}\right)$$

$$I_R = \frac{BH^3}{4}$$

Using parallel axis theorem,

$$I_R = I_G + Ah^2$$

$$I_G = I_R - Ah^2$$

$$= \frac{BH^3}{4} - \frac{1}{2}BH\left(\frac{2}{3}H\right)^2$$

$$I_G = \frac{BH^3}{36}$$

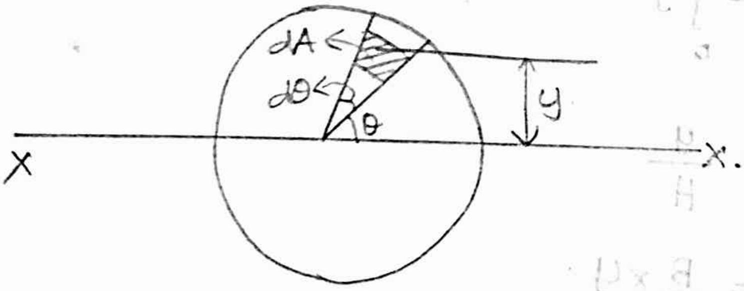
Again using parallel axis theorem,

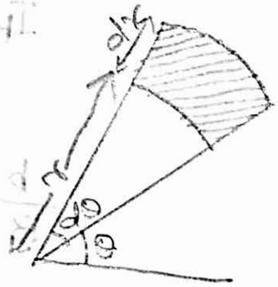
$$I_{PQ} = I_G + Ah^2$$

$$= \frac{BH^3}{36} + \frac{1}{2} \times B \times H \times \left(\frac{H}{3}\right)^2$$

$$I_{PQ} = \frac{BH^3}{12}$$

(c) Moment Of Inertia of a Circular Lamina About an Axis Passing Through Centroid.





$dA = dr . r d\theta$

$dA = r dr d\theta$

$$I_{xx} = \int y^2 dA$$

$$= \int_0^R \int_0^{2\pi} (r\sin\theta)^2 r\,dr\,d\theta$$

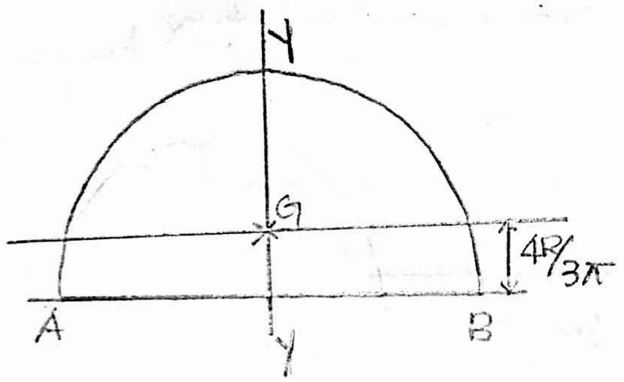
$$= \int_0^R \int_0^{2\pi} r^3 \sin^2\theta\,dr\,d\theta$$

$$= \frac{R^4}{8}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$I_{xx} = \frac{\pi R^4}{4} = \frac{\pi D^4}{64}$$

$$\boxed{I_{xx} = I_{yy} = \frac{\pi D^4}{64}}$$

(d) Moment of Inertia of Semicircle



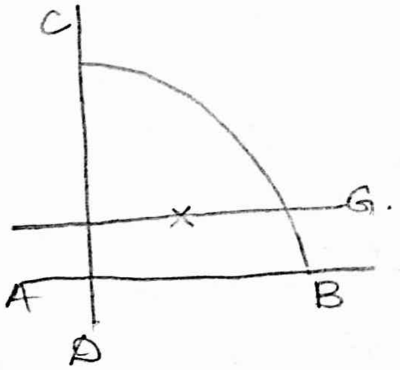
$$I_{AB} = \frac{\pi D^4}{128} = I_{YY}$$

$$I_{GG} = I_{AB} - Ah^2$$

$$= I_{AB} - A\left(\frac{4R}{3\pi}\right)^2$$

$$= 0.11 R^4$$

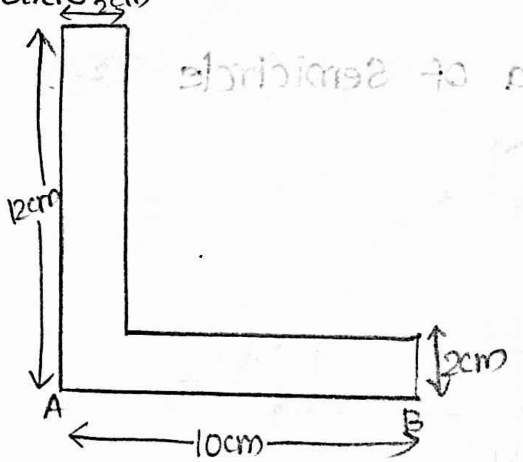
(e) Moment of inertia of Quadrant of a Circle



$$I_{AB} = \frac{\pi D^4}{256} = I_{CD}$$

$$I_{GG} = 0.055 R^4$$

Q: Find the moment of inertia of the area about its centroidal axis.

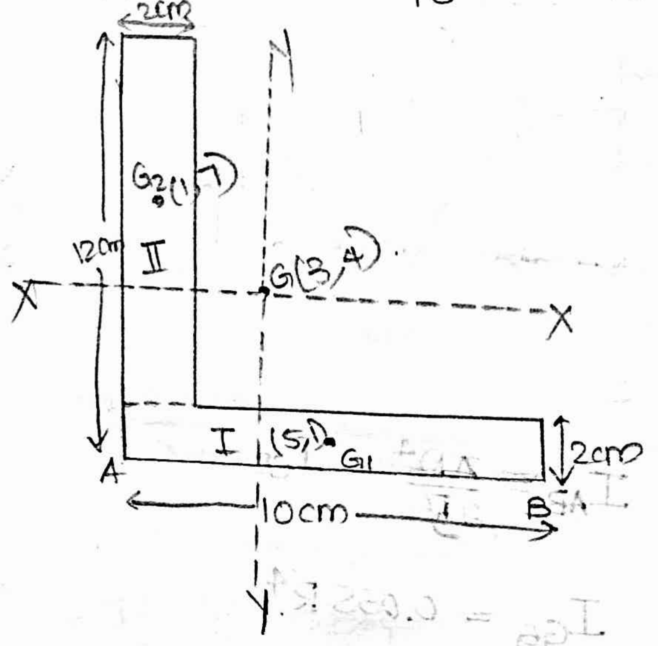


$$A_1 = 20\,cm^2 \qquad x_1 = 5\,cm \qquad y_1 = 1\,cm$$

$$A_2 = 20\,cm^2 \qquad x_2 = 1\,cm \qquad y_2 = 7\,cm$$

$$\bar{x} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2} = \frac{10\times2\times5 + 10\times2\times1}{40} = \underline{\underline{3\,cm}}$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2} = \frac{20\times1 + 20\times7}{40} = \underline{\underline{4\,cm}}$$



$$I_{XX} = I_{XX1} + I_{XX2}$$

$$= (I_{GG_1} + A_1h_1^2) + (I_{GG_2} + A_2h_2^2)$$

$$= \left(\frac{10 \times 2^3}{12} + 20 \times 3^2\right) + \left(\frac{10^3 \times 2}{12} + 20 \times 3^2\right)$$

$$= 186.67 + 346.67$$

$$= \underline{\underline{533.34 \text{ cm}^4}}$$

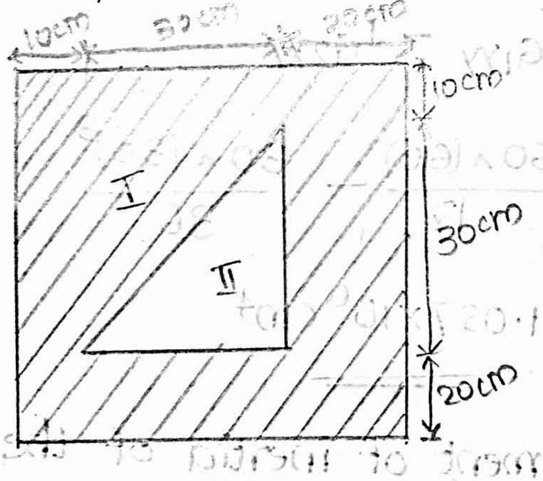
$$I_{yy} = I_{yy1} + I_{yy2}$$

$$= (I_{GG1} + A_1 h_1^2) + (I_{GG2} + A_2 h_2^2)$$

$$= \left[\frac{10 \times 2^3}{12} + 20(2)^2\right] + \left(\frac{2 \times 10^3}{12} + 20 \times 2^2\right]$$

$$=$$

Q: Calculate the moment of inertia of shaded area about centroidal axes I_{Gxx} and I_{Gyy}.

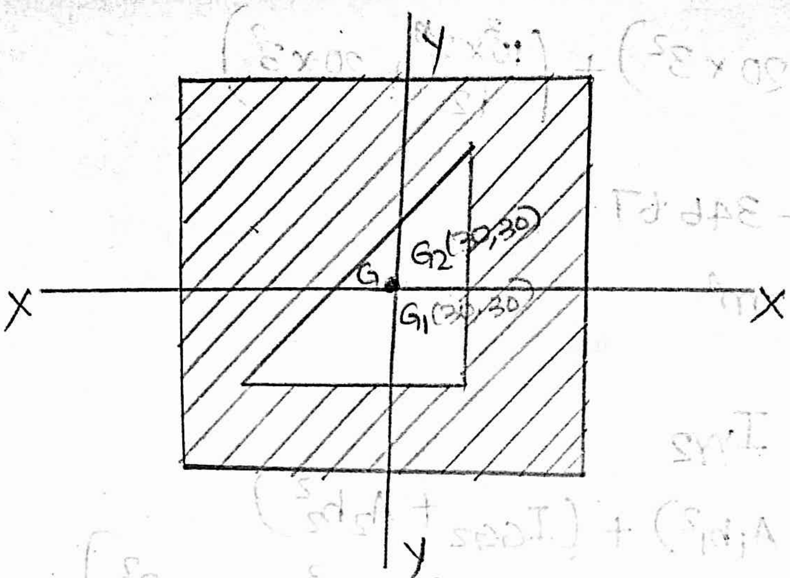


$A_1 = 3600 \text{ cm}^2 \quad x_1 = 30 \text{ cm} \quad y_1 = 30 \text{ cm}$

$A_2 = 450 \text{ cm}^2 \quad x_2 = 30 \text{ cm} \quad y_2 = 30 \text{ cm}$

$$\bar{x} = \frac{A_1 x_1 - A_2 x_2}{A_1 - A_2} = \frac{3600 \times 30 - 450 \times 30}{3600 - 450} = 30$$

$$\bar{y} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2} = \frac{3600 \times 30 - 450 \times 30}{3600 - 450} = \underline{\underline{30}}$$



$I_{Gxx} = I_{G1xx} - I_{G2xx}$

$= \frac{60 \times (60)^3}{12} - \frac{30 \times (30)^3}{36}$

$= \underline{\underline{1.057 \times 10^6 \ cm^4}}$

$I_{Gyy} = I_{G1yy} - I_{G2yy}$

$= \frac{60 \times (60)^3}{12} - \frac{30 \times (30)^3}{36}$

$= \underline{\underline{1.057 \times 10^6 \ cm^4}}$

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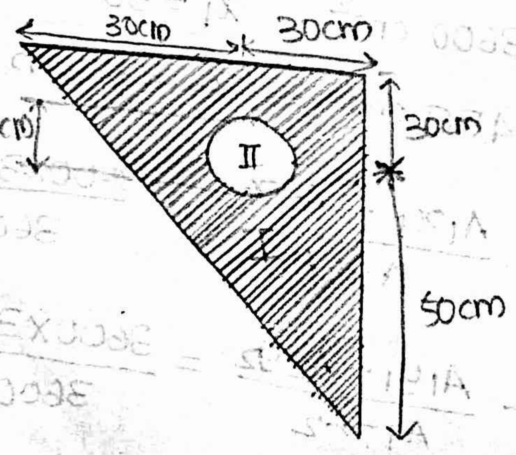
Q: Calculate the moment of inertia of the shaded area with respect to the centroidal axes.

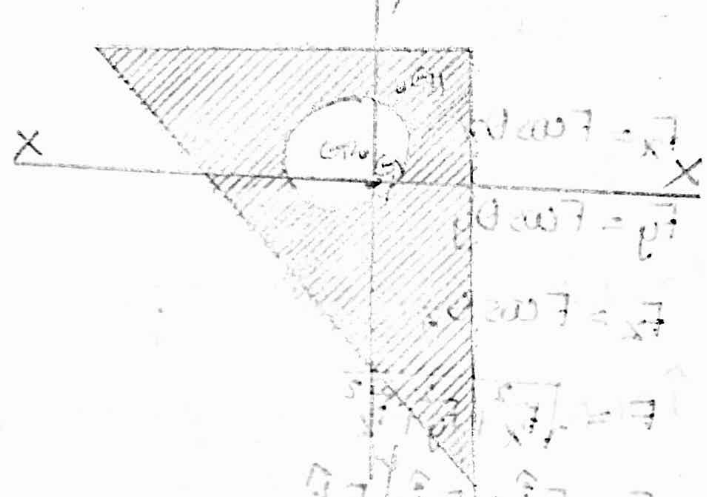


$A_1 = \frac{1}{2} \times 60 \times 80 = 2400\,cm^2 \quad x_1 = 40\,cm \quad y_1 = \frac{2}{3} \times 80 = 53.33\,cm$

$A_2 = \pi\left(\frac{15}{2}\right)^2 = 176.715\,cm^2 \quad x_2 = 30\,cm \quad y_2 = 50\,cm$

$$\bar{x} = \frac{A_1 x_1 - A_2 x_2}{A_1 - A_2} = \frac{2400 \times 40 - 176.715 \times 30}{2400 - 176.715} = \underline{\underline{40.79\,cm}}$$

$$\bar{y} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2} = \frac{2400 \times 53.33 - 176.715 \times 50}{2400 - 176.715} = \underline{\underline{53.598\,cm}}$$



$$I_{GXX} = (I_{GXX1} + A_1 h_1^2) - (I_{GXX2} + A_2 h_2^2)$$

$$= \left(\frac{60 \times 80^3}{36} + \frac{1}{2} \times 60 \times 80 \times (0.265)^2\right) - \left(\frac{\pi \times 15^4}{64} + \frac{\pi \times 15^2}{2 \times 2} (3.598)^2\right)$$

$$= 853333.333 + 168.54 - 2485.049 - 2287.677$$

$$= \underline{\underline{848729.147\ cm^4}}$$

$$I_{GYY} = (I_{GYY1} + A_1 h_1^2) - (I_{GYY2} + A_2 h_2^2)$$

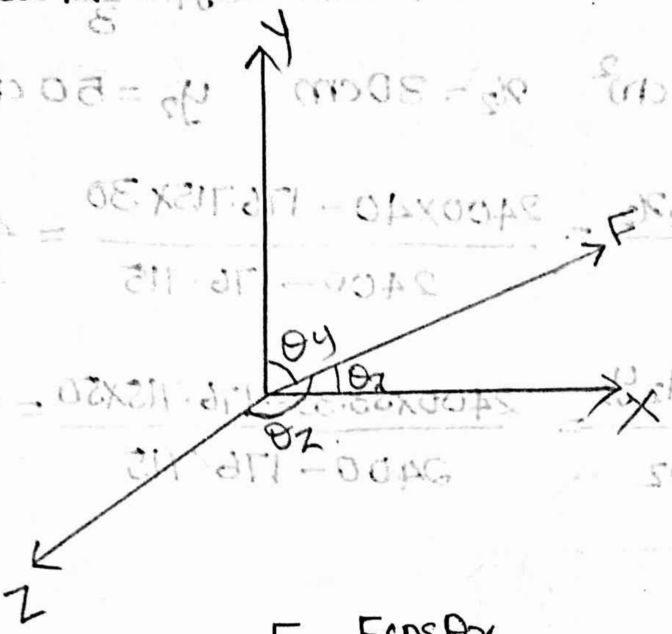
$$= \left(\frac{80 \times 60^3}{36} + \frac{1}{2} \times 60 \times 80 \times (0.795)^2\right) - \left(\frac{\pi \times 15^4}{64} + \pi \times \left(\frac{15}{2}\right)^2 \times (10.795)^2\right)$$

$$= 480000 + 1516.86 - 2485.049 - 20592.909$$

$$= \underline{\underline{458438.902\ cm^4}}$$

18/10/2019

FORCES IN SPACE



$$F_x = F\cos\theta_x$$

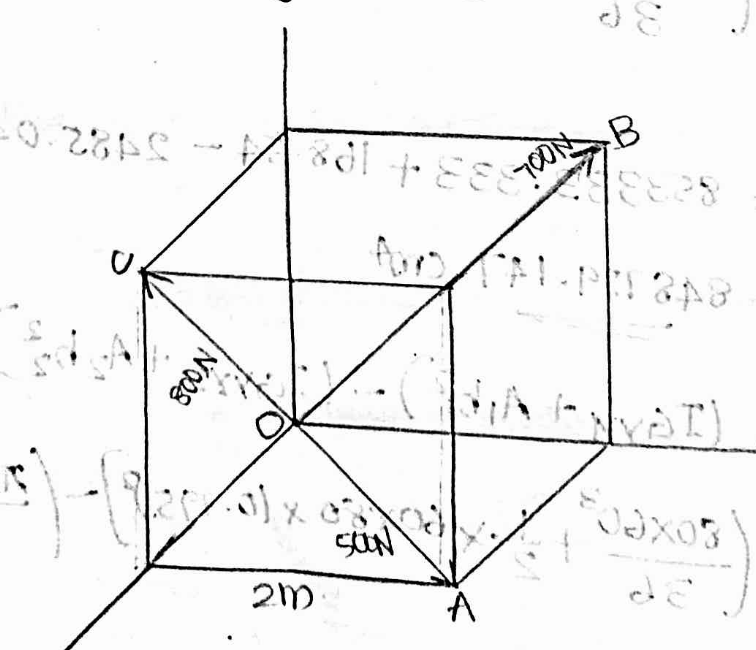
$$F_y = F\cos\theta_y$$

$$F_z = F\cos\theta_z$$

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$F = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

Q: Three forces 500N, 700N and 800N are acting along with three diagonals of adjacent faces of a cube of side 2m as shown in figure. Find the resultant force.



$O(0,0,0)$
$A(2,0,2)$
$B(2,2,0)$
$C(0,2,2)$

$$\text{Unit vector along } OA = \frac{(x_A - x_O)\hat{i} + (y_A - y_O)\hat{j} + (z_A - z_O)\hat{k}}{\sqrt{(x_A-x_O)^2 + (y_A-y_O)^2 + (z_A-z_O)^2}}$$

$$= \frac{2\hat{i} + 2\hat{k}}{\sqrt{4+4}}$$

$$= \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}$$

$$F_{OA} = 500\left[\frac{1}{\sqrt{2}}\hat{i} + 0\hat{j} + \frac{1}{\sqrt{2}}\hat{k}\right] = 353.55\hat{i} + 0\hat{j} + 353.55\hat{k}$$

$$F_{OB} = 700\left[\frac{2\hat{i} + 2\hat{j} + 0\hat{k}}{2\sqrt{2}}\right] = 495\hat{i} + 495\hat{j} + 0\hat{k}$$

$$F_{OC} = 800\left[\frac{0\hat{i} + 2\hat{j} + 2\hat{k}}{2\sqrt{2}}\right] = 0\hat{i} + 565.69\hat{j} + 565.69\hat{k}$$

$$R = F_{OA} + F_{OB} + F_{OC}$$

$$R = 848\hat{i} + 1060\hat{j} + 919\hat{k}$$

$$R = \sqrt{(848)^2 + (1060)^2 + (919)^2} = 1639.29\ N$$

$F_x = 848\ N$

$F_y = 1060\ N$

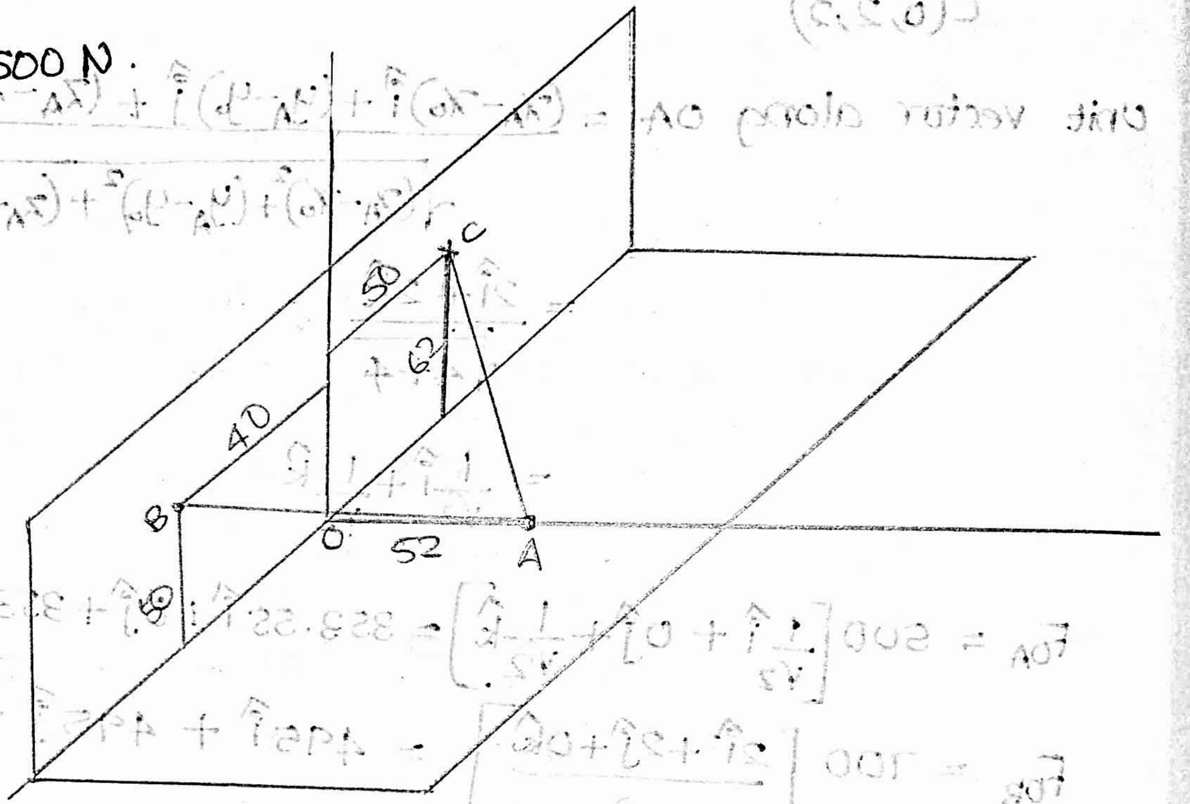
$F_z = 919\ N$

$$\theta_x = \cos^{-1}\left(\frac{F_x}{F}\right) = 58.85^\circ$$

$$\theta_y = \cos^{-1}\left(\frac{F_y}{F}\right) = 49.71^\circ$$

$$\theta_z = \cos^{-1}\left(\frac{F_z}{F}\right) = 55.902^\circ$$

Q: Two cables AB and AC are attached at A as shown in figure. Determine the resultant of forces exerted at A by the two cables, if tension in AB is 2000 N and AC is 1500 N.



$T_{AB} = 2000 N$

$T_{AC} = 1500 N$

A (52, 0, 0)

B (0, 50, 40)

C (0, 62, -50)

$$F_{AB} = 2000 \left[\frac{-52\hat{i} + 50\hat{j} + 40\hat{k}}{82.49} \right]$$

$$= -1260.76\hat{i} + 1212.27\hat{j} + 969.81\hat{k}$$

$$F_{AC} = 1500 \left[\frac{-52\hat{i} + 62\hat{j} - 50\hat{k}}{95.12} \right]$$

$$= -820.02\hat{i} + 977.71\hat{j} - 788.48\hat{k}$$

$$R = F_{AB} + F_{AC} = -2080.78\hat{i} + 2189.98\hat{j} + 181.33\hat{k}$$

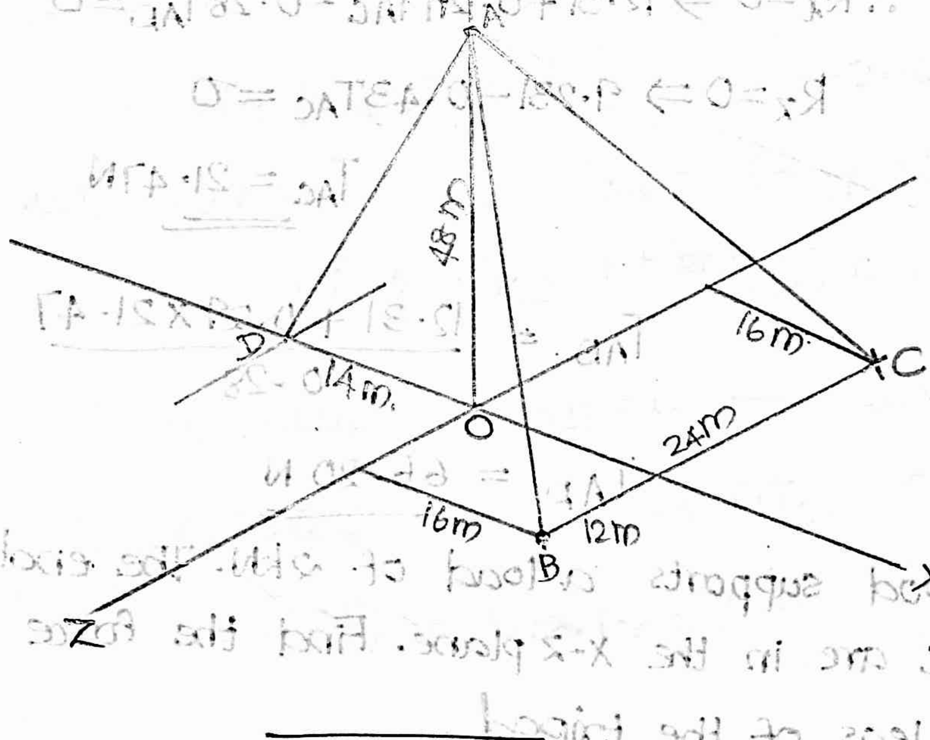
$$R = \sqrt{F_{AB}^2 + F_{AC}^2} = \underline{\underline{3026.31\,N}}$$

$$\theta_x = \cos^{-1}\left(\frac{R_x}{R}\right) = 133.44^\circ$$

$$\theta_y = \cos^{-1}\left(\frac{R_y}{R}\right) = 43.64^\circ$$

$$\theta_z = \cos^{-1}\left(\frac{R_z}{R}\right) = \underline{\underline{86.56^\circ}}$$

Q: A post is held in vertical position by three cables AB, AC and AD. If tension in cable AB is 40N, calculate the required tension in AC and AD so that the resultant of the three forces applied at A is vertical.



A(0,48,0)

B(16,0,12)

C(16,0,-24)

D(-14,0,0)

$$F_{AB} = 40\left(\frac{16\hat{i}-48\hat{j}+12\hat{k}}{52}\right) = 12.31\hat{i}-36.92\hat{j}+9.231\hat{k}$$

$$F_{AC} = T_{AC}\left(\frac{16\hat{i}-48\hat{j}-24\hat{k}}{56}\right) = 0.29T_{AC}\hat{i}-0.86T_{AC}\hat{j}-0.43T_{AC}\hat{k}$$

$$F_{AD} = T_{AD}\left(\frac{-14\hat{i}-48\hat{j}+0\hat{k}}{50}\right) = -0.28T_{AD}\hat{i}-0.96T_{AD}\hat{j}+0\hat{k}$$

$$R = F_{AB}+F_{AC}+F_{AB}$$

$$R_x\hat{i}+R_y\hat{j}+R_z\hat{k} = (12.31+0.29T_{AC}-0.28T_{AD})\hat{i}-(36.92+0.43T_{AC}+0.96T_{AD})\hat{j}+(9.231-0.43T_{AC})\hat{k}$$

Given R is vertical.

$$\therefore R_x=0 \Rightarrow 12.31+0.29T_{AC}-0.28T_{AD}=0$$

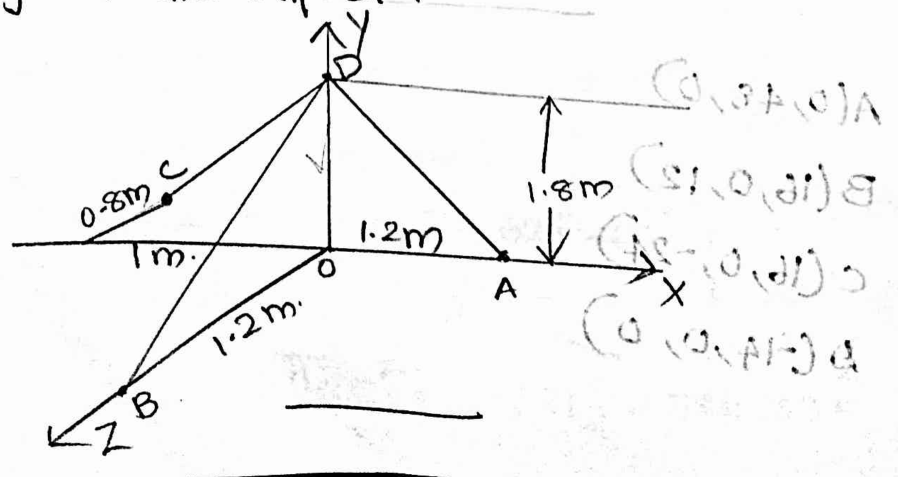
$$R_z=0 \Rightarrow 9.231-0.43T_{AC}=0$$

$$\underline{T_{AC} = 21.47N}$$

$$T_{AD} = \frac{12.31+0.29\times21.47}{0.28}$$

$$\underline{\underline{T_{AD} = 66.20\ N}}$$

Q: A tripod supports a load of 2kN. The ends A, B and C are in the x-z plane. Find the force in the three legs of the tripod.



A (1.2, 0, 0)

B (0, 0, 1.2)

C (−1, 0, −0.8)

D (0, 1.8, 0)

Unit vectors along

$$AD = \frac{-1.2\hat{i}+1.8\hat{j}+0\hat{k}}{2.16} = -0.56\hat{i}+0.83\hat{j}+0\hat{k}$$

$$BD = \frac{0\hat{i}+1.8\hat{j}-1.2\hat{k}}{2.16} = 0\hat{i}+0.83\hat{j}-0.56\hat{k}$$

$$CD = \frac{\hat{i}+1.8\hat{j}+0.8\hat{k}}{2.21} = 0.45\hat{i}+0.81\hat{j}+0.36\hat{k}$$

$$R = F_{AD}+F_{BD}+F_{CD}-2\times10^3\hat{j} = 0$$

$$T_{AD}(-0.56\hat{i}+0.83\hat{j}+0\hat{k}) + T_{BD}(0\hat{i}+0.83\hat{j}-0.56\hat{k})$$

$$+T_{CD}(0.45\hat{i}+0.81\hat{j}+0.36\hat{k}) - 2\times10^3\hat{j} = 0$$

$$(-0.56T_{AD}+0.45T_{CD})\hat{i} + (0.83T_{AD}+0.83T_{BD}+0.81T_{CD}-2000)\hat{j}$$

$$+(-0.56T_{BD}+0.36T_{CD})\hat{k} = 0$$

$$-0.56T_{AD}+0.45T_{CD} = 0 \longrightarrow (1)$$

$$0.83T_{AD}+0.83T_{BD}+0.81T_{CD} = 2000 \longrightarrow (2)$$

$$-0.56T_{BD}+0.36T_{CD} = 0 \longrightarrow (3)$$

$$(1) \Rightarrow T_{AD} = \frac{0.45}{0.56}T_{CD}$$

$$(3) \Rightarrow T_{BD} = \frac{0.36}{0.56}T_{CD}$$

$$\therefore (2) \Rightarrow 0.83\times\frac{0.45}{0.56}T_{CD} + 0.83\times\frac{0.36}{0.56}T_{CD} + 0.81T_{CD} = 2000$$

$$2.0105\, T_{CD} = 2000$$

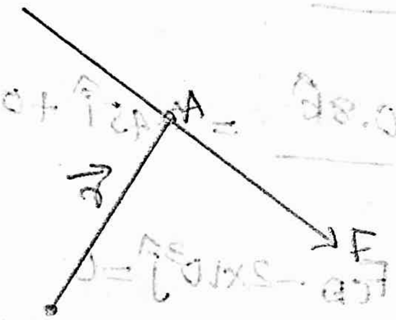
$$T_{CD} = \underline{\underline{994.76\ N}}$$

$$T_{AD} = 799.36\,N$$

$$T_{BD} = \underline{\underline{639.49\ N}}$$

29/10/2019

MOMENT OF A FORCE IN SPACE.



$$M = \vec{r} \times \vec{F}$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

$$\vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

$\vec{r}$ is the position vector of point on line of action of force with respect to the reference point.

Q: A force $\vec{F} = 2\hat{i} + 3\hat{j} - 4\hat{k}$ is applied at point $B(1,-1,2)$. Find moment of force about a point $A(2,-1,3)$.

$$\vec{r} = (x_B - x_A)\hat{i} + (y_B - y_A)\hat{j} + (z_B - z_A)\hat{k}$$

$$= -\hat{i} + 0\hat{j} - \hat{k}$$

$$\vec{M} = \vec{r} \times \vec{F}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 0 & -1 \\ 2 & 3 & -4 \end{vmatrix}$$

$$= 3\hat{i} - 6\hat{j} - 3\hat{k}$$

$$M = \sqrt{M_x^2 + M_y^2 + M_z^2}$$

$$= \sqrt{9+36+9}$$

$$= \sqrt{54} = 7.41 \text{ Nm}$$

Q: A force of 60kN passes through point A(0,2,3), B(7,0,5). Find moment of this force about a point C(7,4,3).

Unit vector along AB $= \dfrac{7\hat{i} - 2\hat{j} + 2\hat{k}}{\sqrt{57}}$

$$F_{AB} = \left(\frac{7\hat{i} - 2\hat{j} + 2\hat{k}}{\sqrt{57}}\right) \times 60 = 55.63\hat{i} - 15.89\hat{j} + 15.89\hat{k}$$

$$\vec{r}_1 = 0\hat{i} - 4\hat{j} + 2\hat{k}$$

$$\vec{M} = \vec{r} \times \vec{F}$$

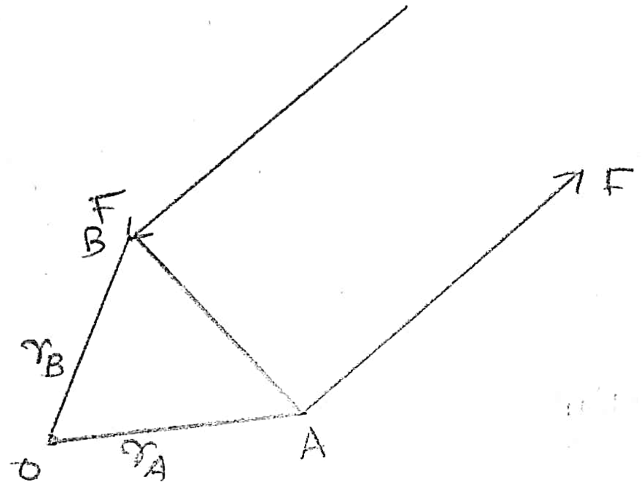
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -4 & 2 \\ 55.63 & -15.89 & 15.89 \end{vmatrix}$$

$$= -31.78\hat{i} + 111.26\hat{j} + 222.52\hat{k}$$

$$M = \sqrt{(31.78)^2 + (111.26)^2 + (222.52)^2}$$

$$= \underline{\underline{250.81 \text{ Nm}}}$$

COUPLE



$$\vec{M_A} = \vec{r_A} \times \vec{F}$$

$$\vec{M_B} = \vec{r_B} \times (-\vec{F})$$

$$\vec{M} = \vec{r_A} \times \vec{F} + \vec{r_B} \times (-\vec{F})$$

$$\vec{M} = (\vec{r_A} - \vec{r_B}) \times \vec{F}$$

$$\vec{M} = \vec{r} \times \vec{F}$$

$$\vec{r} = (x_A - x_B)\hat{i} + (y_A - y_B)\hat{j} + (z_A - z_B)\hat{k}$$

Q: Two forces $F_1 = 50\hat{i} + 80\hat{j} + 100\hat{k}$ and $F_2 = -50\hat{i} - 80\hat{j} - 100\hat{k}$ act at point $A(0.7, 1.5, 1)$ and $B(1, 0.9, -1)$ respectively. Calculate the moment of forces and the perpendicular distance between the forces.

$$\vec{r} = -0.3\hat{i} + 0.6\hat{j} + 2\hat{k}$$

$$\vec{M} = \vec{r} \times \vec{F}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -0.3 & 0.6 & 2 \\ 50 & 80 & 100 \end{vmatrix}$$

$$= -100\hat{i} + 130\hat{j} - 54\hat{k}$$

$$M = \sqrt{(100)^2 + (130)^2 + (54)^2}$$

$$= 172.673 \text{ Nm}$$

$$M = F \times d$$

$$172.673 = 137.477 d$$

$$d = 1.27 \text{ m}$$

Q: Forces 30 kN, 20 kN, 25 kN and 40 kN are concurrent at origin and are directed through points A(2,1,6), B(4,-2,5), C(-3,-2,1) and D(5,1,-20). Determine the resultant of forces:

Unit vector along OA $= \dfrac{2\hat{i}+\hat{j}+6\hat{k}}{\sqrt{4+1+36}}$

$$F_{OA} = \left(\frac{2\hat{i}+\hat{j}+6\hat{k}}{\sqrt{41}}\right) 30$$

$$= 9.37\hat{i} + 4.69\hat{j} + 28.11\hat{k}$$

Unit vector along OB $= \dfrac{4\hat{i}-2\hat{j}+5\hat{k}}{\sqrt{16+4+25}}$

$$F_{OB} = \left(\frac{4\hat{i}-2\hat{j}+5\hat{k}}{\sqrt{45}}\right) 20$$

$$= 11.93\hat{i} - 5.96\hat{j} + 14.91\hat{k}$$

Unit vector along OC = $\dfrac{-3\hat{i}-2\hat{j}+\hat{k}}{\sqrt{9+4+1}}$

$$F_{OC} = \left(\frac{-3\hat{i}-2\hat{j}+\hat{k}}{\sqrt{14}}\right)25$$

$$= -20.04\hat{i} - 13.36\hat{j} + 6.68\hat{k}$$

Unit vector along OD = $\dfrac{5\hat{i}+\hat{j}-20\hat{k}}{\sqrt{25+1+400}}$

$$F_{OD} = \left(\frac{5\hat{i}+\hat{j}-20\hat{k}}{\sqrt{426}}\right)40$$

$$= 9.69\hat{i} + 1.8\hat{j} - 38.76\hat{k}$$

$$F = F_{OA} + F_{OB} + F_{OC} + F_{OD}$$

$$= 10.95\hat{i} - 12.83\hat{j} + 10.94\hat{k}$$

$$R = 10.95\hat{i} - 12.83\hat{j} + 10.94\hat{k}$$

$$R = \sqrt{(10.95)^2 + (12.83)^2 + (10.94)^2}$$

$$= 20.105 \text{ kN}$$

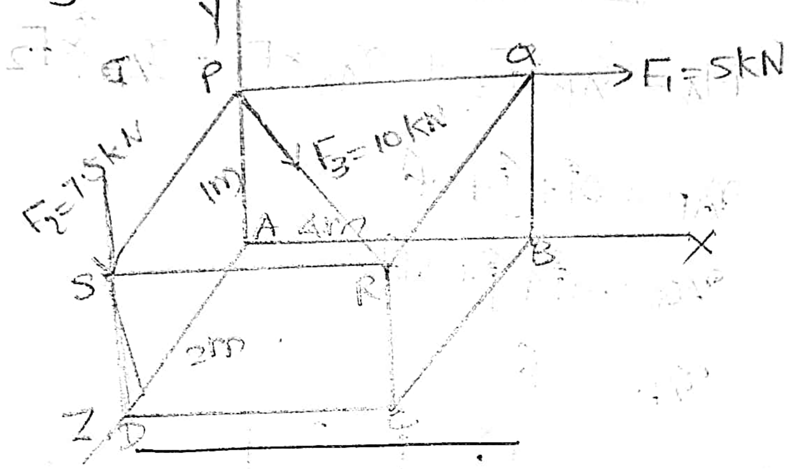
$$\cos\theta_x = \frac{R_x}{R}$$

$$\theta_x = \cos^{-1}\left(\frac{R_x}{R}\right) = 57^\circ$$

$$\theta_y = \cos^{-1}\left(\frac{R_y}{R}\right) = 129.65^\circ$$

$$\theta_z = \cos^{-1}\left(\frac{R_z}{R}\right) = 57.03^\circ$$

Q: A rectangular box is subjected to three forces as shown in figure. Reduce them to an equivalent force-couple system acting at point A.



$P(0,1,0)$ $\quad Q(4,1,0)$ $\quad S(0,1,2)$ $\quad R(4,1,2)$ $\quad D(0,0,2)$

Unit vector along $PQ = \dfrac{4\hat{i}+0\hat{j}+0\hat{k}}{\sqrt{4^2}} = \hat{i}$

$$F_{PQ} = F_1 = \hat{i}(5) = \underline{\underline{5\hat{i}}}$$

Unit vector along $PR = \dfrac{4\hat{i}+0\hat{j}+2\hat{k}}{\sqrt{16+4}} = \dfrac{4\hat{i}+2\hat{k}}{\sqrt{20}}$

$$F_{PR} = F_3 = \left(\frac{4\hat{i}+2\hat{k}}{\sqrt{20}}\right)10$$

$$= \underline{\underline{8.944\hat{i}+0\hat{j}+4.472\hat{k}}}$$

Unit vector along $SD = \dfrac{0\hat{i}-\hat{j}+0\hat{k}}{1} = -\hat{j}$

$$F_{SD} = F_2 = (-\hat{j})7.5$$

$$= \underline{\underline{-7.5\hat{j}}}$$

$$\vec{R} = \vec{F_1}+\vec{F_2}+\vec{F_3}$$

$$= \underline{\underline{13.944\hat{i}-7.5\hat{j}+4.472\hat{k}}}$$

$$R = \sqrt{(13.944)^2 + (7.5)^2 + (4.472)^2}$$

$$= \underline{16.452 \text{ kN}}$$

$$M_A = r_{AP} \times F_3 + r_{AQ} \times F_1 + r_{AD} \times F_2$$

$$r_{AP} = 0\hat{i} + \hat{j} + 0\hat{k}$$

$$r_{AQ} = 4\hat{i} + \hat{j} + 0\hat{k}$$

$$r_{AD} = 2\hat{k}$$

$$r_{AP} \times F_3 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 0 \\ 13.944 & 0 & 4.472 \end{vmatrix} = 4.472\hat{i} - 0\hat{j} - 8.944\hat{k}$$

$$r_{AQ} \times F_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 1 & 0 \\ 5 & 0 & 0 \end{vmatrix} = 0\hat{i} + 0\hat{j} - 5\hat{k}$$

$$r_{AD} \times F_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 2 \\ 0 & -7.5 & 0 \end{vmatrix} = -15\hat{i}$$

$$\therefore M_A = \underline{-10.528\hat{i} + 0\hat{j} - 13.944\hat{k}}$$

MASS MOMENT OF INERTIA

Mass is the quantitative measure of the resistance to change the motion of a body. Inertia of a body is the property by virtue of which it resists any change in its state of rest or of uniform motion. Translatory inertia is defined as mass and rotational inertia is known as moment of inertia.

Mass moment of inertia of a ring of radius R.

Let A be the cross sectional area of a ring and ρ be the mass density of ring material. Consider an elemental length 'dl'.

Volume of ~~the~~ ring of this elemental length $= A \times dl$.

Mass of the elemental volume $= \rho \times A \times dl$.

$$I_{zz} = \int_0^{2\pi R} \rho A \, dl \times R^2$$

$$= R^2 \rho A \left[l \right]_0^{2\pi R}$$

$$= R^2 \rho A \times 2\pi R$$

$$= R^2 \, 2\pi R A \rho$$

$$I_{zz} = M R^2$$

$$I_{zz} = I_{xx} + I_{yy}$$

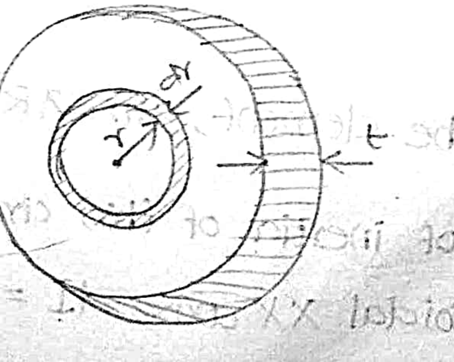
Because of symmetry, $I_{xx} = I_{yy}$

$$I_{zz} = I_{xx} + I_{yy} = 2 I_{xx}$$

$$\boxed{I_{xx} = I_{yy} = \frac{MR^2}{2}}$$

Mass moment of inertia of disc

Consider an elemental ring of radial thickness dr at a distance r from the centre.



Volume of the ring $= 2\pi r dr \times t$

Mass of elemental ring $= 2\pi r dr t \times \rho$

Moment of inertia of the plate about the axis through the centre and perpendicular to the plane of plate is $\int dm\, r^2$.

$$I_{zz} = \int_0^R 2\pi r dr t \rho r^2$$

$$= 2\pi t\rho \int_0^R r^3 dr$$

$$= 2\pi t \rho \left[\frac{r^4}{4}\right]_0^R$$

$$= \frac{2\pi t \rho R^4}{4}$$

$$= (\pi R^2 t \rho)\frac{R^2}{2}$$

$$= \frac{mR^2}{2}$$

Polar moment of inertia, $I_{zz} = I_{xx} + I_{yy} = \frac{mR^2}{2}$

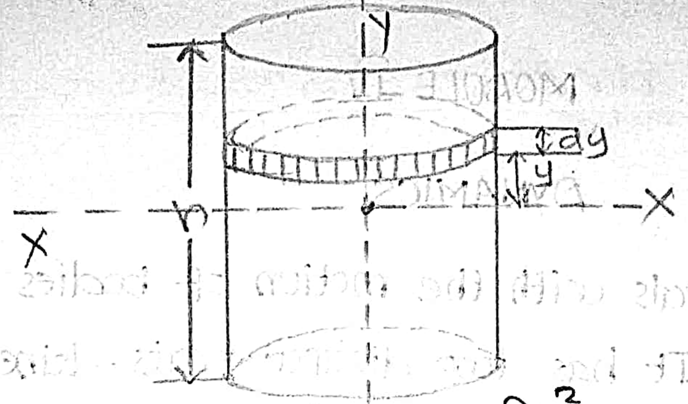
$$\boxed{I_{xx} = I_{yy} = \frac{I_{zz}}{2} = \frac{mR^2}{4}}$$

Mass moment of Inertia of Cylinder.

Consider an elemental circular disc of thickness dy at a distance y from the centroidal XX axis of the cylinder.

Mass of the element, $dm = \pi R^2 dy \rho$

Moment of inertia of thin circular disc about its centroidal XX axis, $dI = \frac{dm R^2}{4}$



$$dI_{xx} = dI + (dm)y^2$$

$$dI_{xx} = \left[\frac{dm\,R^2}{4} + dm\,y^2\right]$$

$$= \pi R^2 dy\,\frac{\rho R^2}{4} + \pi R^2 dy\,\rho y^2$$

$$I_{xx} = \int_{-h/2}^{h/2}\left(\frac{\pi R^4}{4}\rho\right) dy + \pi R^2 \rho \int_{-h/2}^{h/2} y^2 dy$$

$$= 2\pi\frac{R^4}{4}\rho\,[y]_0^{h/2} + 2\pi R^2\rho\left[\frac{y^3}{3}\right]_0^{h/2}$$

$$= \frac{\pi R^4}{2}\,\frac{\rho h}{2} + \frac{2\pi R^2}{3}\rho\left[\frac{h}{2}\right]^3$$

$$= \frac{\pi R^2 h\rho}{4}\left(R^2 + \frac{h^2}{3}\right)$$

$$= \frac{M}{4}\left(\frac{3R^2 + h^2}{3}\right)$$

$$= \frac{M}{12}(3R^2 + h^2)$$

$$\boxed{I_{zz} = I_{xx} = \frac{M}{12}(3R^2 + h^2)}$$